Performance Analysis of a Cooperative Multiple Access Relaying Scheme

Hamid Meghdadi, Vahid Meghdadi, and Jean-Pierre Cances
XLIM/C2S2, University Of Limoges
ENSIL, Parc ESTER, 87068 Limoges, France
Email: [hmeghdadi, meghdadi, cancies]@ensil.unilim.fr

Abstract—The problem of transmitting data from one base station to multiple user mobile stations through some multiple antenna relay stations is studied in this paper. The links between the base station and relays are considered error free and ideal. Therefore, decode and forward strategy is used and we study the downlink between relays and mobile stations. A method to optimize the precoding vector in relays to cancel out multiple access interference and to maximize the signal to noise ratio at the mobile stations is proposed in this paper. We show by Mont-Carlo simulation and a semi-analytic method that maximum diversity advantage can be obtained. We show that the diversity advantage is a simple function of the system parameters: number of relays, number of antennas and the number of users.

I. INTRODUCTION

One of today multimedia transmission challenges is to transmit high data rate digital information to a set of mobile stations in urban highly dense environment. Multiple antenna system has been proposed to overcome the fading random attenuation of the channel and to obtain reliable point to point communications. However, the relative closeness of the antennas make the MIMO (Multiple-Input Multiple-Output) channel correlated. Therefore in a large scale fading case or shadowing, all the MIMO channels can experience deep fading. One promising method to overcome this problem is to use distributed MIMO system or cooperative system, which makes the MIMO channel independent. Since the proposition of this technique, a large number of papers have treated the problem deeply.

In this paper, we are interested in the case where a base station (BS) desires to serve a number of mobile stations (MS) reliably. Therefore a multiuser environment is considered. We use relay stations (RS) installed in the network and suppose that a good high speed link exists between the base station and relay stations. This assumption is quite practical since the BS and RS are considered to be fixed. We can suppose for example that a good optical link is used between BS and RS. This link is considered as the first hop of our channel and is supposed ideal. The relays therefore work in decode and forward strategy. Our problem is now to make the relays cooperate to maximize the signal to noise ratio and to eliminate the multiple access interference (MAI) at each MS. To achieve this purpose, a kind of beamforming is used in relays to make the intended signal for $j$th MS to be sum up coherently and all the other signals to be canceled out at this MS. In [1] the case of beamforming for one base station, two relays and two mobile stations is considered. In this paper, we calculate proper weighting vectors at RS and show by both semi-analytic calculation and Monte-Carlo simulation that our solution achieves the full diversity advantage.

Here, we study two different cases. At first it is supposed that the complete channel state information (CSI) is known at all the RS. In the second case, each RS only knows its own CSI. In [2] the same problem is solved using Lagrange multipliers, which gives a flexible solution and the performance is verified using Monte-Carlo simulation. Here, using the results of [3], we calculate, by a semi-analytic method, the bit error rate (BER) performance of the system and obtain the coding gain and diversity advantage and show that it is a simple function of the system parameters, which is the main contribution of this paper. The rest of this paper is organized as follows: Section II gives a brief description of system model and introduces the equations that define the system. In section III the precoding (weighting) vectors are calculated. This is done under two assumptions i) complete CSI at all relays and ii) independent relays where each RS knows only its own CSI. Furthermore, the results are generalized to the case of $L$ relay stations. In section IV the system performance is analyzed and an analytical formula us given for the BER at high signal to noise ratio (SNR). This formula is confirmed by Monte-Carlo simulations.

II. SYSTEM MODEL

The system is composed of one base station with $M$ antennas, $L$ relay stations with $R$ antennas each one, and $N$ single-antenna mobile stations. We assume that no direct link exists between the BS and MSs. The system model is presented in Fig.1. When a signal $s = [s_1, s_2, \cdots, s_N]^T$ is to be sent from BS to MSs, at the first hop the signal is transmitted to RSs. The relay stations will decode the received signal and calculate the signal to be sent to MSs by multiplying the received signals by some precoding vectors. The signals $x_i$ are then sent to mobile stations. We assume that the channel state information is known in RSs. On the other hand, the MSs do not need to know the channel coefficients. Since the quality of BS-RS link is usually better than that of RS-MS link, we will focus our study on the second hop where the signal is sent from relay stations to mobile stations. For the simplicity, at the first time we will consider a case with two relay stations, then later under the section III-C, we will generalize our results for the
arbitrary $L$. In Fig.1, $x_i$ is the signals sent by the $i^{th}$ relay station, $h_{ij} \sim CN(0, I_R)$ for $i = 1, 2, j = 1 \cdots N$ denotes the Rayleigh channel coefficient vector of size $1 \times R$ between the $j^{th}$ relay and the $i^{th}$ mobile station, and $y_1$ to $y_N$ are the received signals at MS$_j$. Here we have used the same notations as in [1]:

$$x_1 = \sum_{j=1}^{N} s_j w_j^1, \quad x_2 = \sum_{j=1}^{N} s_j w_j^2$$  \hspace{1cm} (1)

where $w_j^i$ of size $R \times 1$ represents the precoding vectors of the $i^{th}$ relay. The signals $x_i$ are then transmitted to MSs, thus the signal received at $j^{th}$ mobile station can be expressed as:

$$y_j = h_{1j} \cdot x_1 + h_{2j} \cdot x_2 + n_j$$  \hspace{1cm} (2)

$$= h_{1j} \cdot \sum_{k=1}^{N} s_k w_k^1 + h_{2j} \cdot \sum_{k=1}^{N} s_k w_k^2 + n_j, \quad j = 1 \cdots N$$

where $n_j \sim CN(0, N_0)$ denotes the gaussian noise. We desire that each mobile station receives only its own relative data, that is to say MS$_1$ receives only $s_1$, MS$_2$ only $s_2$, and so on. It can be written:

$$\sum_{k \neq j} s_k h_{1j} \cdot w_k^1 + \sum_{k \neq j} s_k h_{2j} \cdot w_k^2 = 0$$  \hspace{1cm} (3)

Here two strategies are possible. i) We may assume that both relays know the complete CSI, that is to say both RS$_1$ and RS$_2$ have the complete knowledge of $h_{ij}$ for all $i$ and $j$. ii) Another assumption is that each relay only knows its own channel coefficients, that is to say RS$_1$ knows only $h_{1j}$ for all $j$ and RS$_2$ knows only $h_{2j}$ for all $j$. The first strategy provides higher degree of freedom (thus higher diversity order) but it requires each relay to be aware of the CSI of other relay, therefore there must be an intercommunication between the relays. On the other hand, the second strategy allows independent relay stations at the cost of a loss in the degree of freedom resulting in lower diversity. The second strategy implies that the summations in (3) independently equal zero:

$$\sum_{k \neq j} s_k h_{1j} \cdot w_k^1 = 0, \quad \sum_{k \neq j} s_k h_{2j} \cdot w_k^2 = 0$$  \hspace{1cm} (4)

It is obvious that (4) is more strict than (3), and consequently offers a lower degree of freedom. If either (3) or (4) are satisfied the received signal at MS$_j$ can be written as:

$$y_j = s_j (h_{1j} \cdot w_j^1 + h_{2j} \cdot w_j^2) + n_j$$  \hspace{1cm} (5)

Since no channel information is supposed at the MSs, the term between the parentheses in (5) must be a positive real number.

III. CALCULATION OF PRECODING VECTORS

In this section we will calculate the precoding vectors that allow a coherent detection and eliminate the multiple access interference at the mobile stations. We will use the following notations:

$$H_i = [ h_{1i}^T \mid h_{2i}^T \mid \cdots \mid h_{Ni}^T ]_{N \times R}, \quad i = 1, 2$$ \hspace{1cm} (6)

$$W_i = [ w_1^i \mid w_2^i \mid \cdots \mid w_N^i ]_{R \times N}, \quad i = 1, 2$$ \hspace{1cm} (7)

Using (6) and (7) we can rewrite (2) as:

$$y = (H_1 W_1 + H_2 W_2) s + n$$ \hspace{1cm} (8)

$$= [ H_1 \mid H_2 ] [ W_1 \mid W_2 ] s + n = H_{N \times 2R} W_{2R \times NS} + n$$

In this equation $y \sim [y_1, y_2, \cdots, y_N]^T$ and $n \sim CN(0, N_0 I_N)$ is the column vector of reception noises. In this paper we assume the same signal to noise ratio at all MSs. A more flexible case where different signal to noise ratios are considered for each MS is treated in [2]. In order to satisfy (3) and assuming that the power of the received signal is the same at all mobile stations we may write:

$$H W s = g s$$ \hspace{1cm} (9)

where $g$ is the system gain. Note that if a matrix $W$ satisfies (9) then every matrix $W' = a W$ will also satisfy (9) with $g' = a g$. For simplicity we will choose a $W'$ for which $g' = 1$: equation (9) becomes $H W' s = s$. One possible solution for $W'$ (the only solution which holds for all $s$) is:

$$H_{N \times 2R} W'_{2R \times N} = I_N$$ \hspace{1cm} (10)

Following the same classification as under section II, there are two possible approaches to deal with (10). First strategy is to assume a complete CSI at both relays (each relay knows not only the channel between itself and the MSs, but also the channel from other relays to MSs). The second strategy is to require independent relays meaning that each relay knows only its own channel to the MSs. In the subsections III-A and III-B we will calculate the precoding vectors corresponding to each of the above assumptions and under subsection III-C we will generalize the results for higher numbers of RSs.

A. Complete CSI at Both Relays

If both relays have the complete channel state information, then the whole system is subject to one matrix equation (10). This equation has an answer if and only if $H$ has full row rank (i.e. $R \geq N/2$). In this case $W'$ is the Moore-Penrose pseudo
inverse of $\mathbf{H}$. Thus $\mathbf{W}'$ can be calculated as a function of $\mathbf{H}$ using

$$
\mathbf{W}' = \mathbf{H}^H (\mathbf{HH}^H)^{-1}
$$

(11)

If the total number of relay antennas ($2\mathcal{R}$) is equal to the number of mobile stations ($\mathcal{N}$), then $\mathbf{H}$ is a square matrix and pseudo inverse reduces to normal inverse and we can use $\mathbf{W}' = \mathbf{H}^{-1}$.

When $\mathbf{W}'$ is found, the relays will scale the precoding vectors such that the sum of the powers of transmitted signals from all relay equals the available transmission power $P$, that is to say:

$$
\mathbf{W} = \frac{\sqrt{P} \mathbf{W}'}{\|\mathbf{W}'\|}
$$

(12)

By substituting (11) and (12) in (8) we will obtain:

$$
y = \frac{\sqrt{P} \mathbf{HH}^H (\mathbf{HH}^H)^{-1}}{\|\mathbf{H}^H (\mathbf{HH}^H)^{-1}\|} \mathbf{s} + \mathbf{n}
$$

$$
y = \frac{\sqrt{P} \mathbf{H}^H (\mathbf{HH}^H)^{-1}}{\|\mathbf{H}^H (\mathbf{HH}^H)^{-1}\|} \mathbf{s} + \mathbf{n}
$$

(13)

As we can see in (13), if $\mathbf{W}$ is calculated according to (12), the received signal at each mobile station will depend only on the data symbol intended to that specific mobile station, thus the system can be seen as $\mathcal{N}$ separate channels each of which transmitting one signal $s_j$.

B. Independent Relays

If we want the relays to be independent, it implies that for $i = 1, 2$, only $\mathbf{H}_i$ may be used to calculate $\mathbf{W}_i$. We must therefore require that each relay independently satisfy:

$$
\mathbf{H}_i \in (\mathcal{N} \times \mathcal{R}) \mathbf{W}_i \in (\mathcal{R} \times \mathcal{N}) = \mathbf{I}_N \quad , i = 1, 2.
$$

(14)

In this case the precoding vectors in each relay can be calculated independently, thus each relay needs only to know its corresponding CSI. However, since both $\mathbf{H}_1$ and $\mathbf{H}_2$ must have full row rank, the number of relay antennas must be equal or greater than the number of mobile stations ($\mathcal{R} \geq \mathcal{N}$). Finally $\mathbf{W}'_1$ and $\mathbf{W}'_2$ can be calculated:

$$
\mathbf{W}'_i = \mathbf{H}_i^H \mathbf{H}_i (\mathbf{H}_i \mathbf{H}_i^H)^{-1} \quad , i = 1, 2
$$

(15)

Then $\mathbf{W}_i$ is simply calculated by

$$
\mathbf{W}_i = \frac{\sqrt{P} \mathbf{W}'_i}{\|\mathbf{W}'_i\|}
$$

(16)

where $\sqrt{P}_i$ is the available transmission power at the $i^{th}$ relay and we have $\sum P_i = P$. By substituting (15) and (16) in (8) we will obtain $y = y_1 + y_2 + n$, where $y_1$ and $y_2$ denote the contribution of the first and the second relays in the signal transmission and are given by $y_i = \frac{\sqrt{P} \mathbf{W}'_i}{\|\mathbf{H}_i^H (\mathbf{H}_i \mathbf{H}_i^H)^{-1}\|} \mathbf{s}$.

In the following subsection we will generalize the above discussions to the case of arbitrary relay number.

C. Generalizing the Results for Arbitrary $L$

The above results can be generalized to the case of $L$ relays. System equations remain unchanged with the only difference that we will have the channel vectors $\mathbf{H}_1$ to $\mathbf{H}_\mathcal{L}$ and the precoding vectors $\mathbf{W}_1$ to $\mathbf{W}_\mathcal{L}$.

For the case that complete CSI is provided to all relays (III-A) $\mathbf{W}$ is calculated using

$$
\mathbf{W} = \frac{\sqrt{P} \mathbf{H}^H (\mathbf{HH}^H)^{-1}}{\|\mathbf{H}^H (\mathbf{HH}^H)^{-1}\|}
$$

(17)

with

$$
\mathbf{H} = [ \mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_\mathcal{L} ]_{\mathcal{N} \times \mathcal{RL}}
$$

$$
\mathbf{W} = [ \mathbf{W}_1^T, \mathbf{W}_2^T, \ldots, \mathbf{W}_\mathcal{L}^T ]^T_{\mathcal{RL} \times \mathcal{N}}
$$

(18)

Here we have $\mathcal{RL}$ antennas that cooperate to send $\mathcal{N}$ independent signals to MSs, in this case the diversity gain under maximum ratio combining would be $\mathcal{RL}$. However in our system we want $\mathcal{N} - 1$ undesired signals to be canceled out at each MS. Therefore, we expect a diversity gain of $\mathcal{RL} - \mathcal{N} + 1$. This result is confirmed by simulations.

For the case that each relay knows only its own CSI (III-B), we can calculate the precoding vector $\mathbf{W}'_i$ of the $i^{th}$ relay as follows:

$$
\mathbf{W}'_i = \mathbf{H}_i^H (\mathbf{H}_i \mathbf{H}_i^H)^{-1}, i = 1, \ldots, \mathcal{L}
$$

(19)

Then the signal at the destination is given by (20).

$$
y = \sum_{i=1}^{\mathcal{L}} y_i + n
$$

(20)

Again, the system may be considered as $\mathcal{N}$ parallel channels each of which transmitting the signal $s_j$ to the $j^{th}$ mobile station $\mathbf{MS}_j$. In this case, $\mathcal{RL}$ antennas cooperate to transmit signals to MSs, but since the relays are independent each relay must separately cancel out $\mathcal{N} - 1$ signal interferences. Thus the diversity order would be $\mathcal{RL} - \mathcal{L}(\mathcal{N} - 1)$. It is obvious that the diversity gain of this strategy is less than the first strategy, however the complexity of system is significantly decreased.

IV. System Performance Analysis

In this section we will give a tight approximation under high SNR for bit error probability (BEP) of the system. Without the loss of generality we may focus our study on the case where all relays have the complete information of channel coefficients. The analysis of the case of independent relays is very similar to this case and will not be covered separately.

A. Average Signal to Noise Ratio

We recall from III-A that the received signal at $j^{th}$ mobile station is given by:

$$
y_j = \frac{\sqrt{P}}{\|\mathbf{H}^H (\mathbf{HH}^H)^{-1}\|} s_j + n_j
$$

(21)
Using a signal constellation for which \( E\{|s|^2\} = 1 \), the signal to noise ratio can be calculated by:

\[
\gamma = \frac{P}{N_0\|H^H (HH^H)^{-1}\|^2} \tag{22}
\]

Since the quantity on the denominator of (22) can be calculated as \( \|H^H (HH^H)^{-1}\|^2 = \text{trace} ((HH^H)^{-1}) \), \( \gamma \) can be calculated as \( \gamma = \frac{P}{N_0 \cdot \text{trace}((HH^H)^{-1})} \).

In order to calculate \( \bar{\gamma} \), we note that when \( \bar{X} \) is large enough compared to \( \sigma_X \), then \( E\{1/X\} \) may be approximated by \( 1/E\{X\} \) [4] [5]. Thus we can write:

\[
\bar{\gamma} \approx \frac{P}{N_0 E\{\text{trace}((HH^H)^{-1})\}} \tag{23}
\]

Finally \( E\{\text{trace}((HH^H)^{-1})\} \) can be calculated as:

\[
E\{\text{trace}((HH^H)^{-1})\} = E \left\{ \sum_{j=1}^{N} \frac{1}{\lambda_j} \right\} = \sum_{j=1}^{N} E \left\{ \frac{1}{\lambda_j} \right\} \tag{24}
\]

where \( \lambda_j \) is an eigenvalue of the Wishart matrix \( HH^H \).

### B. Calculating the BEP

The symbol error probability (SEP) of an M-PSK modulation on a telecommunication link with instantaneous SNR \( \gamma \) may be calculated by [6]:

\[
P_s = \int_{0}^{\infty} 2Q(\sqrt{k\bar{\beta}})P_\beta(\beta) d\beta \tag{25}
\]

In this equation \( \beta = \gamma/\bar{\gamma} \) where \( \gamma \) and \( \bar{\gamma} \) denote respectively the instantaneous and average signal to noise ratio, \( k = 2\sin^2(\pi/M) \) and \( P_\beta(\beta) \) is the probability density function (PDF) of \( \beta \). Fig.2 shows \( P_\beta(\beta) \) in comparison with \( Q(\sqrt{k\bar{\beta}}) \) (top) and the product \( Q(\sqrt{k\bar{\beta}})P_\beta(\beta) \) (bottom) for different values of \( \bar{\gamma} \). It can be seen that for large SNRs, the Q-function tends very quickly to zero. Thus for SEP calculations we may limit ourselves to small values of \( \beta \) where the Q-function has significant values. For small values of \( \beta \), the PDF \( P_\beta(\beta) \) in (25) can be approximated by \( a\beta^t \) with \( t = G_d - 1 \) where \( G_d \) is the diversity gain and \( a \) is a constant that will be determined by the original PDF [3]. Furthermore in (25) we can replace the Q-function by its very tight upper bound approximation:

\[
Q(x) \leq \frac{1}{x\sqrt{2\pi}} e^{-x^2/2} \tag{26}
\]

Thus we can approximate (25) by:

\[
P_s \approx \int_{0}^{\infty} 2 \frac{1}{\sqrt{2\pi k\bar{\beta}}} \frac{1}{\sqrt{\pi k\bar{\beta}}} e^{-\frac{k\bar{\beta}}{2}} a\beta^t d\beta \tag{27}
\]

If we pose \( x = k\bar{\beta}/2 \), using the definition of Gamma-function \( (\Gamma(t) \triangleq \int_{0}^{\infty} x^{t-1} e^{-x} dx) \) we can rewrite (27) as:

\[
P_s \approx \frac{2^{t+1}a}{\sqrt{\pi k\bar{\beta}(t+1)}} \int_{0}^{\infty} x^{t-\frac{1}{2}} e^{-x} dx \tag{28}
\]

\[
= \frac{2^{t+1}a}{\sqrt{\pi k\bar{\beta}(t+1)}} \Gamma(t+\frac{1}{2}) = \frac{2a}{\Gamma(\frac{k\bar{\beta}}{2}+1)} \prod_{l=1}^{t+1} (2l-1) \tag{29}
\]

Finally for Grey coding, BEP can be estimated as: \( P_b \approx \frac{1}{\log_2 M} P_s \).

### V. Simulation Results

This section introduces some simulation results that confirm the equations in the previous sections. All simulations are obtained for QPSK modulation using Monte Carlo method in MATLAB.

We have stated under IV-B that if the diversity gain of a communication system is \( G_d \) then for small values of \( \beta = \gamma/\bar{\gamma} \), the PDF \( P_\beta(\beta) \) may be approximated by \( a\beta^{G_d-1} \). Also we affirmed under III-C that the diversity gain is \( LR - N + 1 \) for the case that all relays have the complete knowledge of CSI and \( L(R - N + 1) \) for the case of independent relays. In Fig.3, the simulated value of \( \log(P_\beta(\beta)) \) is plotted as a function of \( \log(\beta) \) for different numbers of MSs (N), RSs (L), and antennas per RS (R). We can see that our results are confirmed by the simulation. For instance if two independent 3-antenna relays cooperate in signal transmission toward two mobile stations, the diversity gain will be \( 2 \times 3 - 2(2-1) = 4 \). Then \( P_\beta(\beta) \) can be approximated by \( \beta^{4} \), therefore the slope of the curve would be 3 which is confirmed in Fig.3.

As stated under section II, there are two possible scenarios depending on whether or not the relay stations are provided with the knowledge of channel state information of other relays. Figure 4 shows that if the channel information is available to both relays, lower bit error rate (BER) is obtained. This is at the cost of more complexity in the transmission protocol. On the other hand, if each of the relays knows only its respective channel information, the system is more practical at the cost of higher BER. Furthermore, Figure 4 shows that the theoretical expression given in (28) is a very tight approximation of BER for high SNRs.

Figure 5 shows the BER as a function of \( E_s/N_0 \) for different number of relays all for a given number of relay antennas. Having more relay antennas compared to mobile stations results in lower bit error rates. We can see that when the number of mobile stations is more than the number of relay
antennas, an error floor appears in the curves.

Figure 6 depicts the system performance for different number of relay stations. All curves are obtained for 3-antenna relay stations and two mobile stations. The only difference is the number of relay stations contributing in signal transmission. As we can see higher relay numbers results in better system performance.

VI. CONCLUSION

A scheme of multi-antenna multi-relay telecommunication system was addressed. Multiple access interference was canceled out by means of multiplying the signal by proper precoding vectors at relay stations. Precoding vectors were calculated to meet two different requirements. In the first assumption the complete CSI is supposed to be known by all RSs, while in the second one, each relay knows only its own relative channel coefficients. Bit error probability was given by semi-analytic calculations and confirmed by Monte Carlo simulation. We demonstrated that the achieved diversity is $LR - N + 1$ for the first case and $L(R - N + 1)$ for the second case.

REFERENCES


